Grounds for Effective Theories

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Abstract

I propose an account of the physical content, and the non-fundamental character, of effective quantum field theories. The account combines contemporary work on the renormalization group approach with contemporary work on grounding. As I show, the account complements other realist approaches to effective quantum field theories in the literature: it can be used to solve some problems that those other accounts face.

1 Introduction

Quantum field theories—which are among the most empirically well-confirmed theories of science to date—are often taken to be 'effective'. That is, each quantum field theory is often taken to hold only up to some particular, fixed, finite energy. At higher energies, the theory in question ceases to be reliable. It does not accurately describe such high-energy physics; another quantum field theory—of that higher-energy physics—is needed. But at lower energies, like the energies at which experiments are conducted, the theory in question accurately describes the physical facts. In other words, at lower energies, the theory in question is true. Call quantum field theories, understood in this way, 'effective theories'.

Recently, several philosophers have begun developing realist accounts[1] of effective theories (Fraser, 2018; 2020; Wallace, 2006; Williams, 2019; forthcoming). The basic idea, under-

[1]I follow the literature in calling these accounts 'realist'. The word 'realist' is, unfortunately, pretty ambiguous: there are many accounts of what scientific—or quantum field-theoretic—realism is, and they often differ from one another in significant ways. But in this paper, by calling these accounts 'realist',

lying these accounts, is that effective theories are theories of non-fundamental physical facts. They are not theories of the fundamental physical world; they describe non-fundamentalia only.

For various reasons, these accounts often appeal to a cluster of formal techniques and results known as the 'renormalization group approach'. Roughly put, in many physical domains, the renormalization group approach can be used to show the following: extremely different effective theories, at extremely high energies, often generate more-or-less the same sorts of effective theories at lower energies. In other words, the renormalization group approach shows that high-energy physical theories which posit different interactions, different physical structures, and so on, generate low-energy physical theories which posit similar interactions, similar physical structures, and so on.

The renormalization group approach can be used to address some issues that arise for realist accounts of effective theories. For instance, as argued by Fraser (2020), the renormalization group approach can be used to justify various mathematical manipulations which must be performed, in order to formulate effective theories that are well-defined. As argued by Williams (2019), the renormalization group approach can be used to distinguish (i) those features of effective theories which describe real physics, from (ii) those features of effective theories which are mere mathematical artifacts of the formalism.

In this paper, I use the renormalization group approach to continue developing realism about effective theories. To start, I propose an account of effective quantum field theories' physical contents. Basically, the account says that the physical propositions described by an effective theory concern objects and interactions at a particular energy level. Then I show how the renormalization group approach, in conjunction with the metaphysical notion of grounding, can be used to account for the way in which effective theories are non-fundamental. Basically, the account says that the non-fundamental facts described by an effective theory

I simply mean that they take effective theories to be more than mere calculational devices for predicting the behaviors of observables. These realist accounts take effective theories to describe real features of the unobservable physical world. Later on, I will describe—in more detail—what those real features are.

at a low energy are grounded in the non-fundamental facts described by an effective theory at some higher energy; and this fact—that higher-energy facts ground lower-energy facts—is itself grounded in facts concerning structural relationships among all higher-energy effective theories. The renormalization group approach describes those structural relationships.

In Section 2, I provide a brief formal introduction to effective theories and the renormalization group approach. In Section 3, I propose the account of the physical content of effective theories. In Section 4, I propose the account of how effective theories are non-fundamental. In Section 5, I show that this account complements other realist approaches to effective theories, and I use it to solve some problems that have been raised for realist approaches.

2 Effective Theories and the Renormalization Group Approach

Through a series of formal manipulations, quantum field theories defined over all energy scales can be transformed into effective theories; that is, into theories defined only up to particular energy scales. In this section, I briefly explain how. Then I describe the basic ideas underlying the renormalization group approach to analyzing theories like these.

Let $\mathcal{L}$ be a Lagrangian density; so roughly put, $\mathcal{L}$ specifies a particular physical theory. To pick a simple example, suppose that $\mathcal{L}$ is defined as follows.

$$\mathcal{L} = \frac{1}{2}\big(\partial_\mu \phi(x)\big)^2 - \frac{1}{2}m^2\phi(x)^2 - \frac{\lambda}{4!}\phi(x)^4 \tag{1}$$

The function '$\phi(x)$' represents a real scalar field on spacetime: so ϕ assigns a scalar to each spacetime point x. The constants 'm' and 'λ' are called 'coupling constants': they characterize the strengths of the physical processes and interactions that $\mathcal{L}$, as a whole, describes.

$\mathcal{L}$ can be used to define the following important quantity.

$$Z = \int \mathcal{D}\phi e^{-i \int d^4x \mathcal{L}} \tag{2}$$

Z is called a 'transition amplitude'. It can be used to express the probability that the field represented by '$\phi(x)$' will transition from one state to another. In particular, Z describes a field which (i) begins in the ground state at the idealized time $t = -\infty$, (ii) evolves in a quantum-field-theoretic way constrained by the Lagrangian density $\mathcal{L}$, and (iii) ends in the ground state at the idealized time $t = +\infty$. The term '$\mathcal{D}\phi$' represents a measure over all possible states of the quantum field. And altogether, very roughly put, the integral expression in (2) is like a weighted sum over all possible ways for the quantum field to transition from the ground state to the ground state; what counts as a 'possible way' is determined by $\mathcal{L}$. In other words, the integral expression is like a weighted sum of all possible field trajectories—where $\mathcal{L}$ determines which trajectories are and are not possible—that begin and end in the ground state.

Through a series of formal manipulations, the Lagrangian density $\mathcal{L}$ can be transformed into a modified Lagrangian density that describes a different—but related—class of physical facts. In particular, whereas $\mathcal{L}$ describes physical processes and interactions that involve arbitrarily high energies, the modified Lagrangian density does not. Instead, the modified Lagrangian density describes physical processes and interactions whose degrees of freedom involve energies up to, but not beyond, a particular 'cutoff' energy Λ_c. In other words, the modified Lagrangian density is only taken to be accurate at energies below that cutoff. At higher energies, the modified Lagrangian density ceases to apply; it should not be taken seriously; it does not correctly describe the physics there. In this way, the modified Lagrangian density is an 'effective' field theory: it is a field theory, but it is only effective—it is only accurate and reliable—up to a certain energy level.[2] Let '$\mathcal{L}_{\Lambda_c}$' denote this modified Lagrangian density.

For a variety of reasons, $\mathcal{L}_{\Lambda_c}$ does a better job of facilitating calculations than $\mathcal{L}$. Those

[2]For discussion of subtle issues concerning the effectiveness of effective field theories, see Franklin (2020).

reasons are well-rehearsed in both the physics literature[3] and the philosophical literature,[4] so I will not reproduce them in detail here. But roughly put, the basic idea is this. Because of certain features of $\mathcal{L}$, standard methods for calculating Z lead to undefined results. Some integrals, used to calculate quantities that feature in empirical predictions based on Z, diverge. The Lagrangian density $\mathcal{L}_{\Lambda_c}$ does not lead to divergences like these. So $\mathcal{L}_{\Lambda_c}$ does a better job of facilitating calculations.

$\mathcal{L}_{\Lambda_c}$ is obtained by making a series of modifications to the expressions in $\mathcal{L}$. For starters, the field $\phi(x)$ and the couplings m and λ are modified. Let $\phi_{\Lambda_c}(x)$, m_{Λ_c}, and λ_{Λ_c} be this modified field and these modified couplings, respectively. In addition, new interaction terms are added to $\mathcal{L}_{\Lambda_c}$. These terms may feature powers of $\phi_{\Lambda_c}(x)$ that are greater than four.[5]

The modified Lagrangian density $\mathcal{L}_{\Lambda_c}$ can be used to define the following modified transition amplitude.

$$Z_{\Lambda_c} = \int_{\Lambda_c} \mathcal{D}\phi_{\Lambda_c} e^{-i\int d^4x \mathcal{L}_{\Lambda_c}} \tag{3}$$

Z_{Λ_c}, like Z, represents the transition amplitude associated with a field that begins in the ground state and returns to the ground state, all while subject to the constraints imposed by the corresponding Lagrangian density. The integral expression in (3), like the integral expression in (2), is akin to a weighted sum.

The difference between Z_{Λ_c} and Z can be understood in terms of differences in their respective sums. Recall that in (2), the integral is like a weighted sum over *all* possible field trajectories from the ground state to the ground state. In (3), however, the integral is like a weighted sum over *some* possible field trajectories from the ground state to the ground state. The sum in (3), roughly put, conflates fields which differ only in certain high-energy

[3] For accessible summaries of those reasons, and for more thorough discussions of the physics presented in this section, see (Duncan, 2012; Lancaster & Blundell, 2014; Peskin & Schroeder, 1995; Zee, 2010).

[4] See (Fraser, 2020; Li, 2015; Miller, 2018; Ruetsche, 2018; Williams, forthcoming).

[5] Polchinski (1984) contains a thorough analysis of how to modify $\mathcal{L}$ to generate $\mathcal{L}_{\Lambda_c}$. His analysis includes more details of the physics than I am discussing here, such as the equation for the propagator (Polchinski, 1984, p. 275).

regimes: in particular, regimes above the cutoff energy Λ_c. So the transition amplitude Z_{Λ_c} is not sensitive to differences among physical states associated with energies above that cutoff. That is why $\mathcal{L}_{\Lambda_c}$ and Z_{Λ_c} only provide reliable information about physics up to a certain energy level.

The renormalization group approach is a cluster of formal techniques for manipulating $\mathcal{L}_{\Lambda_c}$ and Z_{Λ_c}. Those manipulations yield information about physics at energy scales below Λ_c.[6] When applied to the cutoff Lagrangian density $\mathcal{L}_{\Lambda_c}$ and the transition amplitude Z_{Λ_c}, the renormalization group approach produces a new Lagrangian density and a new transition amplitude. Roughly put, those new expressions describe physical facts at an energy scale which is marginally less than Λ_c. Repeated applications of the renormalization group approach yield theories of physical facts at lower and lower energies. Repeat enough times, and the result is a theory of the physics at the energy levels which experiments can probe.

Here are some details. To start, choose an energy Λ which is less than—though very close to—Λ_c. The single integral in (3) can be split into two smaller integrals, each of which depends on Λ. Formally, the result looks like this.

$$\int_{\Lambda_c} \mathcal{D}\phi_{\Lambda_c} e^{-i\int d^4x \mathcal{L}_{\Lambda_c}} = \int_{\Lambda} \mathcal{D}\phi_{\Lambda} e^{-i\int d^4x \mathcal{L}'} \int_{\Lambda}^{\Lambda_c} \mathcal{D}\phi_h e^{-i\int d^4x \mathcal{L}'_h} \tag{4}$$

The first, left-most integral is just the integral from (3). The second integral '$\int_\Lambda$' sums over fields with low-energy degrees of freedom: in particular, degrees of freedom corresponding to energies less than Λ. The subscript 'Λ' in 'ϕ_Λ' indicates that the integral sum is restricted in this way; the superscript '$'$' in '$\mathcal{L}'$' indicates that this Lagrangian density is defined in terms of ϕ_Λ rather than ϕ_{Λ_c}. The third integral '$\int_\Lambda^{\Lambda_c}$' sums over fields with higher-energy degrees of freedom: in particular, degrees of freedom corresponding to energies between Λ and Λ_c. The subscript 'h' in 'ϕ_h' indicates this high-energy restriction in the integral sum; the superscript '$'$' and subscript 'h' in '$\mathcal{L}'_h$' indicates that this Lagrangian density is defined in terms of both

[6] The books mentioned in footnote 3 cover many of those techniques in detail. The particular renormalization group approach discussed here is based on techniques from (Wilson & Kogut, 1974).

ϕ_Λ and ϕ_h.

The third integral, in (4), cannot be solved exactly. But it can be approximated using techniques from perturbation theory. In particular, it can be expressed as follows.

$$\int_{\Lambda}^{\Lambda_c} \mathcal{D}\phi_h e^{-i\int d^4x \mathcal{L}'_h} = e^{-i\int d^4x \delta\mathcal{L}'} \tag{5}$$

The expression '$\delta\mathcal{L}'$' represents an incremental change in $\mathcal{L}'$. So (5) says that the third integral in (4) can be re-expressed as an exponential whose exponent contains an integral of an incremental change in $\mathcal{L}'$.

Along with some rescalings—of both parameters and fields—(3), (4), and (5) can be used to derive the following expression for Z_{Λ_c}.

$$Z_{\Lambda_c} = \int_{\Lambda} \mathcal{D}\phi_\Lambda e^{-i\int d^4x \mathcal{L}_\Lambda} \tag{6}$$

Note that $\mathcal{L}_\Lambda$ is equal to the sum of $\mathcal{L}'$ and $\delta\mathcal{L}'$. That is, $\mathcal{L}_\Lambda$ is a Lagrangian density obtained by incrementally modifying the Lagrangian density $\mathcal{L}'$ which corresponds to low-energy degrees of freedom.

Furthermore, note that $\mathcal{L}_\Lambda$ is only defined up to a certain energy level: namely, Λ. In other words, $\mathcal{L}_\Lambda$ is only taken to be accurate at energies of Λ or lower. At higher energies, $\mathcal{L}_\Lambda$ ceases to apply; it should not be taken seriously. So $\mathcal{L}_\Lambda$, like $\mathcal{L}_{\Lambda_c}$, is an effective theory.

Let us take stock of what these formal expressions establish. Roughly put, (3) says that Z_{Λ_c} is obtained by summing over field configurations which (i) are constrained by $\mathcal{L}_{\Lambda_c}$, and (ii) involve energies up to Λ_c. As the above series of formal manipulations show, (3) can be transformed into (6). And the integral in (6) is a sum over field configurations too: namely, those configurations which (i) are constrained by $\mathcal{L}_\Lambda$, and (ii) involve energies up to Λ.

In moving to a slightly lower energy scale, the Lagrangian density has changed. For the Lagrangian density at the lower energy scale is $\mathcal{L}_\Lambda$ rather than $\mathcal{L}_{\Lambda_c}$. The transition amplitude remains the same: that is what (6) says. But the Lagrangian density—the physical theory at the new energy scale—is different.

In order to describe the physics at an even lower energy scale Λ', simply repeat the above procedure. Split the integral in (6) into two smaller integrals: one sums over field configurations associated with low-energy degrees of freedom, while the other sums over field configurations associated with high-energy degrees of freedom. Calculate the latter integral. Then simplify. The result will be an integral expression for Z_{Λ_c} that sums over field configurations which (i) are constrained by another modified Lagrangian density, call it '$\mathcal{L}_{\Lambda'}$', and (ii) involve energies only up to Λ'.

By iterating the above procedure, one obtains a series of Lagrangian densities that describe physical facts at lower and lower energy scales. Various renormalization group equations express the relationship between changes in energy scales and corresponding changes in the relevant Lagrangian densities. For instance, certain functions—called 'beta functions'—describe how the values of coupling constants vary with incremental changes in the energy scale.

Think of these equations as describing how shifts in energy induce a 'flow' through the space of possible Lagrangian densities. $\mathcal{L}_{\Lambda_c}$ represents one candidate starting point. The above procedure, applied once, moves us from $\mathcal{L}_{\Lambda_c}$ to $\mathcal{L}_{\Lambda}$. This new Lagrangian density describes physical facts at a new, lower energy scale. Repeated applications of the above procedure carry us to Lagrangian densities describing physical facts at lower and lower energies. In this way, the renormalization group approach traces out paths through the 'theory-space' of Lagrangian densities. And renormalization group equations describe this 'renormalization group flow' along these paths.

Remarkably, several formal results suggest that in many empirically relevant cases, this flow converges to a subspace of Lagrangian densities that are empirically tractable. Many Lagrangian densities, outside that subspace, are not empirically tractable in the requisite ways. They contain terms which have infinitely many coupling constants, all of which have a significant effect on the empirical quantities that they would be used to calculate.[7] But in many cases, the renormalization group flow leads to Lagrangian densities which are more amenable to empirical investigation. Only a finite number of coupling constants, in these Lagrangian densities, have a non-negligible effect on the empirical quantities that experiments can probe. And these Lagrangian densities also describe physical facts at energy scales to which we have empirical access.

For example, Polchinski showed that given a large space of scalar theories $\mathcal{S}$, the renormalization group flow leads to a finite-dimensional subspace $\mathcal{F}$ of $\mathcal{S}$ (1984). The space $\mathcal{F}$ consists of Lagrangian densities for scalar fields with specific properties: for instance, their coupling constants are small. The renormalization group transformations, on which Polchinski focused, preserve various important empirical quantities; just as the transformations described in (4)–(6) preserve transition amplitudes. The Lagrangian densities in this subspace take the form of, and include, the Lagrangian density described by (1). As a result, at the low energies in which we perform experiments, basically any scalar field theory looks like a theory with the form of that Lagrangian density.

To summarize, there are at least two important, extremely impressive achievements of the renormalization group approach. First, the renormalization group approach provides an account of how theories that describe physical facts at lower energies can be extracted from theories that describe physical facts at higher energies. In particular, the approach describes how Lagrangian densities defined at higher energies, like Λ_c, may be transformed into Lagrangian densities defined at lower energies, like Λ. Second, the renormalization group approach suggests that this transformation ultimately leads to theories which are empirically tractable. In other words, this transformation generates empirically testable Lagrangian densities at empirically accessible energy scales.

[7] For instance, these couplings characterize interactions which make non-negligible contributions to the calculated values of scattering amplitudes, correlation functions, and so on.

3 The Physical Content of Effective Theories

In this section, I present an account of the physical content of effective theories. The account focuses on the physical contents of the Lagrangian densities discussed in Section 2. It specifies the physical facts which those Lagrangian densities describe.

Here is the account. Basically, it says that an effective quantum field theory—at a particular energy scale—describes the physical objects, physical properties, and so on, which are associated with physical phenomena at that scale.

> EFFECTIVE CONTENT
>
> For each energy level Λ_x and each effective theory $\mathcal{L}_{\Lambda_x}$ at that energy level, the physical propositions described by $\mathcal{L}_{\Lambda_x}$ are propositions about quantum fields, particles composed of quanta, physical interactions among fields and particles, transition amplitudes, correlation functions, and so on, at that energy level.

In other words, $\mathcal{L}_{\Lambda_x}$ describes physical propositions like "The probability of the field transitioning from this state to that state is such-and-such," and "This particle interacts in thus-and-so ways with this other particle." These physical propositions only concern quantum fields, particles, physical interactions, transition amplitudes, correlation functions, and so on, at the specific energy level Λ_x. So these fields, particles, and so on, are not fundamental. Nevertheless, they still exist.

Plenty of these physical propositions are false, of course. For only some effective theories obtain at the actual world. Those effective theories, that actually obtain, describe physical facts that actually hold. In other words, those effective theories describe physically real fields, physically real particles, physically real interactions, physically real transitions, and so on. The other effective theories describe fields, particles, interactions, transitions, and so on, that are merely possible.

According to EFFECTIVE CONTENT, the physical facts described by a true effective theory $\mathcal{L}_{\Lambda_x}$ are akin to the sorts of physical facts that non-fundamental special sciences describe. For instance, take evolutionary biology. Rabbits and wolves interact with one another in accord, roughly, with the Lotka-Volterra equations.[8] The biological facts describing those interactions concern phenomena at a particular energy level: the level corresponding to medium-sized dry goods. So to put it roughly: rabbits and wolves interact at this level. They do not interact at other energy levels. The physical facts describing their interactions concern phenomena at this particular energy level only.

Similarly for effective theories. A true effective theory $\mathcal{L}_{\Lambda_x}$ describes facts like "The probability of the field transitioning from this state to that state is such-and-such." These facts concern objects and phenomena—in particular, physical field states and transition probabilities—at the particular energy level Λ_x. Other physical facts described by $\mathcal{L}_{\Lambda_x}$ concern particles—specifically, certain sorts of quanta—at that energy level, and how they interact. So $\mathcal{L}_{\Lambda_x}$ describes physically real fields, physically real particles, physically real interactions, physically real transitions, and so on, all at the specific energy level Λ_x.

According to Effective Content, for any given energy level, many physical objects and interactions—many fields, particles, transitions, and so on—produce phenomena at that level. There are facts about field transitions, particle interactions, and so on, at one energy level. There are facts about field transitions, particle interactions, and so on, at a slightly higher energy level. There are facts about field transitions, particle interactions, and so on, at a slightly lower energy level. And according to Effective Content, this proliferation of fields and particles—and facts about them—continues across the energy hierarchy.

To understand this hierarchical picture of the physical content of effective theories, consider analogous hierarchies among special sciences. Theories in evolutionary biology describe facts about objects and interactions at the energy level of medium-sized dry goods. For instance, as mentioned above, some theories describe facts about predatory-prey interactions between rabbits and wolves. Theories in molecular biology describe facts about objects and interactions at the energy level of molecules. For instance, some theories describe facts about

[8]These equations describe how the sizes of predator populations and prey populations change over time.

transcriptional interactions between ribosomes and mRNA. Theories in astrophysics describe facts about objects and interactions at the galactic energy level. For instance, some theories describe facts about the gravitational interaction between the Milky Way and Andromeda.

These special sciences, like effective quantum field theories, form a hierarchy across different energy levels. The tables below illustrate how. The first table lists the objects, interactions, and energy levels that are described by molecular biology, evolutionary biology, and astrophysics.

Theory	Objects	Interactions	Energy
Molecular Biology	Ribosomes, mRNA	Transcription	Higher
Evolutionary Biology	Wolves, Rabbits	Predator-Prey	Medium
Astrophysics	Galaxies	Gravitational	Lower

The second table lists the objects, interactions, and energy levels that are described by different effective quantum field theories.

Theory	Objects	Interactions	Energy
$\mathcal{L}_{\Lambda_1}$	Fields$_1$, Particles$_1$	Transitions$_1$, Correlations$_1$	Λ_1 (Higher)
$\mathcal{L}_{\Lambda_2}$	Fields$_2$, Particles$_2$	Transitions$_2$, Correlations$_2$	Λ_2 (Lower)
$\mathcal{L}_{\Lambda_3}$	Fields$_3$, Particles$_3$	Transitions$_3$, Correlations$_3$	Λ_3 (Even Lower)

So just as molecular biology, evolutionary biology, and astrophysics describe different objects interacting in different ways at different energy levels, effective theories like $\mathcal{L}_{\Lambda_1}$, $\mathcal{L}_{\Lambda_2}$, and $\mathcal{L}_{\Lambda_3}$ describe different objects interacting in different ways at different energy levels too.

This approach, to the physical content of effective theories, raises a question: exactly what is the relationship between the different objects and different interactions described by effective theories at different energies? According to Effective Content, physical reality contains a vast smorgasbord of fields, particles, transitions, correlations, and so on, all drawn from different energy levels. So a natural question is: how do all those different elements of physical reality, from all those different energy levels, relate? Are they completely unrelated to one another? Or are they connected in some way?

In what follows, I answer these questions. The physical objects and processes from different energy levels are, I will claim, intimately related to one another. The metaphysical relation of grounding can be used to describe that relationship. In particular, grounding can be used to account for the relationship between (i) the physical facts described by true effective theories at higher energies, and (ii) the physical facts described by true effective theories at lower energies. Let us see how.

4 The Grounds of Effective Theories

In this section, I present an account of how effective theories are grounded. The account relies on the relationships—among Lagrangian densities at different energies—discussed in Section 2. Basically, the account says that facts about those relationships help to ground the physical facts that effective theories, at lower energies, express.

By way of preparation, it is worth making some comments about grounding. There are many debates over specific features of grounding: for example, there are debates over which formal conditions it satisfies (Correia & Skiles, 2019; Fine, 2012; Schaffer, 2016). But for the purposes of this paper, two specific features of grounding—about which there is lots of agreement—will be especially relevant: grounding is *explanatory*, and grounding relates[9] the *more fundamental* to the *less fundamental*. Let us consider each of these features in turn.

First, the explanatory feature: grounding is an explanatory relation. In particular, if one fact grounds another, then the former provides a metaphysical explanation of the latter.

[9] In this paper, I follow Rosen (2010) in taking grounding to be a relation between facts. Other approaches—like those endorsed by Fine (2012) or Schaffer (2016)—could be used instead, in the account of effective theories to come. That account would just need to be reworded.

For instance, take a standard example: the existence of Socrates grounds the existence of the singleton set containing Socrates (Fine, 1994). Because Socrates' existence grounds the existence of the set, it follows that the existence of the set is explained by Socrates' existence.

Second, the fundamentality feature: grounding relates more fundamental facts to less fundamental facts. That is, if one fact grounds another, then the former is more fundamental than the latter. For instance, if facts about physical brain states ground facts about mental states (Schaffer, 2016), then the brain state facts are more fundamental than the mental state facts. So grounding relates different levels of the world. It captures the way in which less fundamental facts derive from, or are generated by, more fundamental facts.

In what follows, two types of grounding will be relevant: partial grounds, and full grounds. A full ground of a fact is a ground which, on its own, suffices for that fact. For example, suppose that a fact about a physical brain state fully grounds a fact about a certain corresponding mental state. Then the former fact is completely sufficient for the latter fact to obtain. A partial ground of a fact is a ground which, on its own, may or may not suffice for that fact. For example, plausibly, the existence of Socrates only partially grounds the existence of the singleton set containing Socrates. For Socrates' existence is not—on its own—sufficient for the existence of the set: in order for the set to exist, various other set-theoretic facts must obtain too.

With that as background, let us turn to the account of how effective theories are grounded. Let $\mathcal{T}$ be a reasonably natural space of theories; the sort of space that physicists study. So for instance, $\mathcal{T}$ could be the space—analyzed by Polchinski—of Lagrangian densities for scalar fields with specific properties. Or $\mathcal{T}$ could consist of Lagrangian densities for QED, or Lagrangian densities for the standard model. Regardless, however, suppose that $\mathcal{T}$ supports a renormalization group flow which preserves physically measurable quantities: transition amplitudes, correlation functions, and so on. And let Λ be the energy at which scientists measure those empirical quantities.

In addition, suppose that the following three facts obtain. First, at energy level Λ, $\mathcal{L}_\Lambda$

correctly describes the physical facts. Call the collection of these facts, which $\mathcal{L}_\Lambda$ describes, 'Λ-Scale'. Second, for some extremely high energy Λ', there is a theory $\mathcal{X}_{\Lambda'}$ in $\mathcal{T}$ such that (i) $\mathcal{X}_{\Lambda'}$ correctly describes the physical facts at that high energy, and (ii) the renormalization group flow carries $\mathcal{X}_{\Lambda'}$ to $\mathcal{L}_\Lambda$. Let 'High-Energy' represent the collection of facts which $\mathcal{X}_{\Lambda'}$ describes. Third, for all theories at extremely high energies, the renormalization group flow carries those theories to a finite-dimensional surface of attraction in $\mathcal{T}$ of which $\mathcal{L}_\Lambda$ is a member; and $\mathcal{X}_{\Lambda'}$, in particular, gets carried to $\mathcal{L}_\Lambda$. Call this fact 'Surface'.

Now for the complete account of the grounds of effective quantum field theories. It is given below.

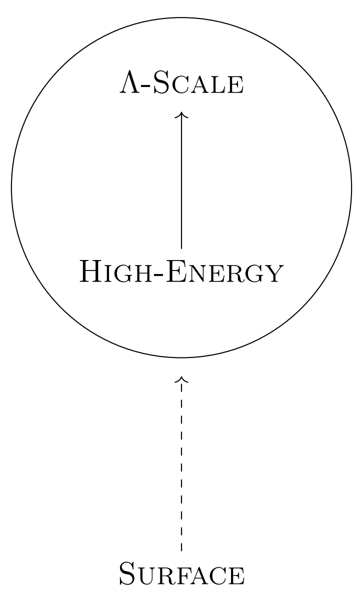
Effective Grounds

(1) High-Energy fully grounds Λ-Scale.

(2) Surface partially grounds the fact that High-Energy fully grounds Λ-Scale.

Roughly put, (1) says the following: the physical facts described by effective theories at lower energies are grounded in the physical facts described by effective theories at higher energies. And roughly put, (2) says the following: that fact about grounding—expressed by (1)—is itself grounded in facts about the structure of the space of effective theories.

Here is Effective Grounds in picture form.



The solid arrow from High-Energy to Λ-Scale represents the fact that the former fully grounds the latter. The dashed arrow from Surface to the circle represents the fact that the grounding fact within the circle—the fact that High-Energy fully grounds Λ-Scale—is itself partially grounded in Surface. So altogether, Effective Grounds says the following: the physical facts described by $\mathcal{L}_\Lambda$ are grounded in the physical facts described by some specific effective theory at an extremely high energy, and that grounding relationship is itself grounded in more general facts about renormalization group flow in $\mathcal{T}$.

In two different ways, Effective Grounds provides an account of how less fundamental physical facts arise. First, according to (1), the physical facts accurately described by $\mathcal{L}_\Lambda$—at the fairly low energy Λ—are grounded in physical facts at a higher energy, described by a particular higher-energy theory in $\mathcal{T}$.[10] As discussed in Section 3, all of these physical facts concern fields, particles, interactions, transitions, and so on, occurring at specific energy levels. So (1) says that facts about fields, particles, and so on, at energy Λ, are grounded in facts about fields, particles, and so on, at some higher energy. In other words, the high-energy facts 'generate' the facts described by $\mathcal{L}_\Lambda$ in the sense that the renormalization group flow carries the theory describing the former facts to the theory describing the latter facts.

Second, according to (2), the ground-theoretic relationship described by (1) is itself grounded in a fact about the general structure of all candidate effective field theories at higher energies. In particular, the grounding connection between $\mathcal{L}_\Lambda$ and some specific high-energy theory is itself grounded in the fact that all higher-energy physical facts—of the sort described by higher-energy theories in $\mathcal{T}$—generate lower-energy physical facts of more-or-less the same structure as the physical facts described by $\mathcal{L}_\Lambda$. 'More-or-less the same structure' means, to a first approximation, that all of these lower-energy facts are described by theories with roughly the same mathematical form. In particular, those theories—with roughly the

[10]In what follows, I sometimes switch between two different ways of summarizing Effective Grounds. According to one summary, Effective Grounds concerns the grounds for *the physical facts described by* $\mathcal{L}_\Lambda$. According to the other summary, Effective Grounds concerns the grounds for *the truth of* $\mathcal{L}_\Lambda$ *itself*. For present purposes, the differences between these two characterizations of Effective Grounds are largely irrelevant. Take the latter to be a heuristic gloss on the former; or vice versa, if you prefer.

same mathematical form—all belong to the same finite-dimensional surface in $\mathcal{T}$.

Effective Grounds provides a distinctively and uniquely realist view of the formal, mathematical techniques that comprise the renormalization group approach. For according to Effective Grounds, those techniques do more than just describe mathematical relationships among various formal theories. Those techniques also describe a certain worldly relation that physical reality instantiates. That worldly relation connects physical phenomena at different energy scales. It runs from the facts described by certain high-energy theories to the facts described by certain low-energy theories. And in particular, according to Effective Grounds, that worldly relation is the relation of grounding.

In other words, Effective Grounds reifies the formal, mathematical relationships—among different effective theories—that the renormalization group approach describes. The renormalization group approach, recall, consists of a series of mathematical methods for extracting lower-energy formalisms from higher-energy formalisms. So Effective Grounds says that those mathematical methods describe mathematical relations which have real, physical, worldly correlates: namely, instances of the grounding relation.

Effective Grounds suggests the following picture of effective field theories. Renormalization group flow generates many different paths through the theory-space of Lagrangian densities. One of those paths—call it the 'actual path'—is actual: in other words, the Lagrangian densities along the actual path are true effective theories of the actual physical facts at particular energy levels in our world. Within the actual path, the Lagrangian densities at higher energies fully ground the Lagrangian densities at lower energies; this is basically what (1) says. And the renormalization group flow helps to ground all that. In other words, the renormalization group flow partially grounds the fact that along the actual path, higher-energy theories fully ground lower-energy theories; this is basically what (2) says.

According to the picture under consideration here, the renormalization group approach is a tool for uncovering ground-theoretic relationships among many different possible effective theories. Each path through theory-space, which the renormalization group flow traces out, corresponds to a possible world: in particular, a possible world whose physics—at various energy levels—is the physics described by corresponding Lagrangian densities along the path in question. In other words, each path describes which high-energy effective theories—at some possible world—ground which low-energy effective theories at that same possible world. So the renormalization group approach is more than a cluster of formal techniques for extracting information about actual-world physics at low energies. The renormalization group approach is also a guide to the metaphysical facts of the matter: it can be used to uncover the way that more fundamental theories ground less fundamental theories across a broad region of possibility space.

This, by the way, is why (2) says that Surface *partially* grounds—rather than *fully* grounds—the fact that High-Energy fully grounds Λ-Scale. For Surface does not say anything about which path, in that possibility space, is actual. Surface merely describes certain structural relations that obtain among all possible effective theories at particular energies. Surface does not say anything about which of those effective theories, at which energies, are true. So Surface, taken on its own, is not sufficient to ground the fact that High-Energy fully grounds Λ-Scale. The full grounds of the latter, ground-theoretic fact include facts about which path—through possibility space—actually obtains.

In the facts High-Energy and Surface, I took $\mathcal{T}$ to be a *natural* space of theories. By 'natural', I simply meant that the Lagrangian densities in $\mathcal{T}$ are unified, objectively similar, non-gerrymandered, and more generally, apt targets for scientific investigation; in short, $\mathcal{T}$ is the sort of space that physicists study. Because of this, $\mathcal{T}$ is capable of featuring in good scientific explanations of phenomena.

I took $\mathcal{T}$ to be natural because otherwise, Effective Grounds would imply that facts about highly unnatural spaces ground—and so explain—the truth of $\mathcal{L}_\Lambda$. But unnatural spaces are not the sorts of spaces that feature in good scientific explanations: they are too disunified and gerrymandered to be explanatory. So facts about those spaces cannot serve as grounds for Λ-Scale.[11]

Effective Grounds is quite different from other accounts of how facts described by physical theories are grounded. The literature on grounding generally focuses on relatively simple cases from physics.[12] For instance, while listing some illustrative examples of grounding, Fine writes that facts about particles' accelerations are grounded in facts about the forces acting on those particles (2012, pp. 38-39). While discussing the ways in which laws ground regularities, Rosen focuses on cases from Newtonian gravitation (2010, pp. 119-120). And while providing his own illustrative examples of grounding, Schaffer focuses on non-relativistic Bohmian mechanics (2021, pp. 175-176).[13] Effective Grounds, in contrast, focuses on more up-to-date cases from contemporary particle physics. In that respect, Effective Grounds represents an improvement over these other 'toy cases' of the grounds of physical facts.

Note that Effective Grounds accounts for how the renormalization group approach plays an explanatory role in theoretical and empirical physics.[14] Those approaches have been used to established facts that explain the success of theories like $\mathcal{L}_\Lambda$. And those explanations are built into Effective Grounds's account of the facts expressed by effective theories: in particular, those explanations are captured by the ground-theoretic link between Surface and the fact that High-Energy grounds Λ-Scale. In other words, since grounding is an explanatory relation, Effective Grounds implies that Surface partially explains why certain low-energy theories are grounded in certain high-energy theories.

[11]For instance, let $\mathcal{T}$ be the space of Lagrangian densities analyzed by Polchinski, and let $\mathcal{T}^*$ be $\mathcal{T}$ with some arbitrary, scattered collection of Lagrangian densities removed. Then the Lagrangian densities in $\mathcal{T}^*$ are extremely disunified and gerrymandered; T^* is not the sort of space, in other words, that scientists study. And so $\mathcal{T}^*$ does not feature in good scientific explanations of phenomena.

[12]One notable exception is McKenzie, who explores some ways of using grounding to formulate versions of ontic structural realism (2014; 2020).

[13]Metaphysicians tend to present simplistic cases like these because, for the most part, metaphysicians have been interested in theorizing about grounding in domains other than contemporary particle physics. For instance, they have been concerned with how complex logical formulas are grounded in simpler logical formulas (Fine, 2012), or how normative facts are grounded in physical facts (Rosen, 2010), or how mental states are grounded in physical states (Schaffer, 2021). The present paper, I hope, shows that effective quantum field theories constitute another domain in which grounding can do interesting philosophical work. Metaphysicians would do well to study those theories.

[14]For more on the explanatory capacities of the renormalization group approach, see (Reutlinger, 2014).

In closing, here is a summary of Effective Grounds. The physical facts described by effective theories at low energies are fully grounded in the physical facts described by effective theories at high energies. Moreover, that fact—about the ground-theoretic relationship between low-energy theories and high-energy theories—is itself partially grounded in facts about renormalization group flow. The low-energy physical world, in other words, is generated by the high-energy physical world. And that generation is partially explained by the theory of renormalization.

5 Realist Approaches to Effective Theories

Together, Effective Content and Effective Grounds comprise a theory—call it 'Grounding Theory'—of the physical content, and non-fundamentality, of effective quantum field theories. In this section, I explain how Grounding Theory complements other realist accounts of effective quantum field theories in the literature. I also show that Grounding Theory can be used to avoid some problems that have been raised for those realist accounts.

Here is a core component of various realist accounts of effective theories: the formal structures in an effective theory that have real physical content are precisely those structures which are "invariant across independent...choices about how to model the physics at [higher energies] where the theory is empirically inapplicable" (Williams, 2019, p. 220). In other words, if some formal structure of some low-energy theory is invariant—in the sense that it is generated by higher energy theories—then plausibly, that formal structure is not a mere mathematical artifact of the low-energy theory. Rather, that formal structure describes real features of the physical world at that low energy.

Grounding Theory respects all that. Many of the physical facts described by $\mathcal{L}_\Lambda$ are invariant, in the sense of 'invariant' used by Williams; for they are among the facts which feature in Λ-Scale. According to Grounding Theory, those physical facts are grounded in whichever higher-energy theory actually holds at energies well above Λ. And according to Grounding Theory, that grounding relationship is itself grounded in the invariance of the renormalization group flow. So if some formal structure of $\mathcal{L}_\Lambda$ is invariant—in the sense that it is generated by higher-energy theories in $\mathcal{T}$—then according to Grounding Theory, that formal structure is not a mere mathematical artifact. That formal structure describes real features of the physical world at energy Λ.

In addition, Grounding Theory addresses an issue mentioned by Fraser (2020). In order to substantiate the claim that effective theories capture "unobservable aspects of the world, ... we need a precise characterization of the non-fundamental descriptive content of QFTs" (Fraser, 2020, p. 289). For absent a precise characterization of that content, realist approaches to effective theories seem pretty unattractive. At best, those approaches are underdeveloped: they claim that effective theories are non-fundamental, but they do not explain how. And at worst, Fraser argues, those approaches are inferior to empiricism: better to adopt an empiricist account of an unobservable posit, he suggests, than to be a realist about that posit without any such account at all (2020, pp. 289-290).

Grounding Theory addresses this issue. In particular, Grounding Theory provides an account of the non-fundamental content of effective theories. For according to Effective Content, the physical content of a true effective theory at energy Λ consists of facts about non-fundamental bits of physical reality at Λ: fields, particles, transition amplitudes, correlation functions, interactions, and so on. According to Effective Grounds, those facts are grounded in High-Energy; and Surface explains why. That is how effective theories have non-fundamental physical content: they describe physical facts – about fields, particles, and so on – at lower energies, that are grounded in physical facts – about other fields, other particles, and so on – at higher energies. And facts about renormalization group flow, within theory-space, explain why those grounding connections obtain.

One might object that for the purposes of providing a realist account of effective field theories, Grounding Theory goes too far. In order for fans of the renormalization group approach to be realists about effective theories, one might claim, they need not commit to realism about theories at higher energies. They need only commit to realism about low energy theories: the theories, in particular, at energies which experiments can probe. In other words, one might claim, they need not endorse HIGH-ENERGY. They need not claim that some effective theory correctly describes the physical facts at some high energy scale.

My response: the view endorsed by this objection—call it 'LOW-ENERGY REALISM'—is unattractive. On the one hand, LOW-ENERGY REALISM says that all higher-energy theories are false; we should not be realists about them. On the other hand, LOW-ENERGY REALISM says that we should be realists about some low-energy theory $\mathcal{L}_\Lambda$ *because* the renormalization group flow carries all high-energy theories to low-energy theories with the same basic structure as $\mathcal{L}_\Lambda$. And that is a strange combination of views. For if all higher-energy theories are false, then how could facts about their behavior under the renormalization group flow explain, or justify, realism about $\mathcal{L}_\Lambda$? Why does it matter that these false theories behave this way or that under some mathematical operations, like those which comprise the renormalization group approach? There are plenty of mathematical operations that transform plenty of false theories into true ones. Those operations do not explain, or justify, realism about those true theories. So if every higher-energy theory is false, then it is unclear how the renormalization group approach could explain—or justify—realism about $\mathcal{L}_\Lambda$.

Because of all this, I prefer GROUNDING THEORY to LOW-ENERGY REALISM. GROUNDING THEORY explains realism about low-energy theories by appealing to realism about higher-energy theories; that is basically what EFFECTIVE GROUNDS says. LOW-ENERGY REALISM, however, offers no such explanations. And that is a strike against it.

It is worth mentioning one other virtue of GROUNDING THEORY: it can be used to avoid a problem that Ruetsche poses for realist approaches to effective theories. Ruetsche argues that according to those approaches, the mechanisms and structures in effective theories describe "nature's deep and hidden springs, springs that act to bring about the phenomena the theory explains" (2018, p. 1176). For instance, according to Ruetsche, realist interpretations of $\mathcal{L}_\Lambda$ are committed to fields acting on a fundamental, four-dimensional spacetime (2018, p. 1188). The problem for realism, Ruetsche observes, is that spacetime could turn out to be quite different: it could be 26-dimensional and stringy.

To see how Grounding Theory can be used to avoid this problem, note that the theories invoked in Grounding Theory only make claims about the physical world up to particular energy levels. The physical facts described by $\mathcal{L}_\Lambda$, for instance, only concern objects, interactions, and phenomena at energy Λ. Of course, the mathematical field ϕ_Λ which features in $\mathcal{L}_\Lambda$ is defined on a four-dimensional spacetime. But it does not follow that, according to realist interpretations of $\mathcal{L}_\Lambda$, spacetime is *fundamentally* four-dimensional. What follows is that there exists a non-fundamental physical object—namely, a non-fundamental spacetime—which (i) has four dimensions, and (ii) figures in interactions that occur *at energy level* Λ. According to Grounding Theory, the existence of that non-fundamental spacetime is grounded in physical facts about phenomena at energies above Λ, and that grounding relationship obtains because of facts about renormalization group flow. This, of course, is consistent with *all* these physical facts, at *all* these energy levels, ultimately being grounded in facts about a fundamental, 26-dimensional, stringy spacetime.

So there is lots to like about Grounding Theory. It complements extant realist approaches to effective quantum field theories. It fills in some lacunae for those realist approaches, by providing an illuminating account of effective theories' non-fundamental physical content. And it can be used to avoid some problems that realist approaches tend to face.

6 Conclusion

Grounding Theory provides a precise account of the physical content, and the non-fundamentality, of effective quantum field theories. According to Effective Content, the physical content of an effective theory at a particular energy level consists of propositions which describe fields, particles, interactions, transitions, correlations, and so on, at that energy level. According to Effective Grounds, the physical facts described by effective theories at lower energies are grounded in the physical facts described by effective theories at higher energies, and facts about renormalization group flow explain why. In addition, Grounding Theory is quite attractive: it can be used to fill several gaps in realist approaches to effective theories, and it solves some problems that those approaches face. So realists about quantum field theories would do well to accept Grounding Theory.

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